Exercise 38

Use continuity to evaluate the limit.

$$\lim_{x \to 4} 3^{\sqrt{x^2 - 2x - 4}}$$

Solution

Apply Theorem 8 to bring the limit inside the exponential function. This theorem applies because the exponential function is continuous at 2, the limit of the inner function as $x \to 4$.

$$\lim_{x \to 4} 3^{\sqrt{x^2 - 2x - 4}} = 3^{\lim_{x \to 4} \sqrt{x^2 - 2x - 4}}$$

Apply Theorem 8 again to bring the limit inside the square root function. This theorem applies because the square root function is continuous at 4, the limit of the inner function as $x \to 4$.

$$\lim_{x \to 4} 3^{\sqrt{x^2 - 2x - 4}} = 3^{\sqrt{\lim_{x \to 4}(x^2 - 2x - 4)}}$$

$$= 3^{\sqrt{(4)^2 - 2(4) - 4}}$$

$$= 3^{\sqrt{4}}$$

$$= 3^2$$

$$= 9$$